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SOLUTIONS OF PROBLEMS IN NO. 8.

Solutions of problems in No. 8 have been received as follows:

From Prof. A. B. Evans, 34, 35 & 36; R. M. DeFrance, Esq., 34; Henry Gunder, 34 & 35; L. Regan, 34 & 35; James Stott, 35; and Walter Siverly, 35 & 36.

35. "Given $xyz = 18$, (1); $x^2 - y^2 + z^2 = 33$, (2); $(x^2 - yz)^3 + (y^2 - xz)^3 + (z^2 - xy)^3 - 3(x^2 - yz)(y^2 \quad xz\,(z^2 - xy) = 6561$, (3); to find x, y and z."

SOLUTION BY R. M. DEFRANCE, ESQ., MERCER, PA.

Put $x + y + z = s$ and $xy + xz + yz = m$, and substitute in (3) and it becomes $(s^3 - 3ms + 54)^2 - 108(s^3 - 3ms + 54) = 3645$; whence we get $s^3 - 3ms = \pm 81$ ....(4). By substitution (2) becomes $s^2 - 2m = 33$ ....(5). Eliminating m from (4) and (5) we get $s^2 - 99s = \mp 162$. Multiplying this eq. by s we have $s^4 - 99s^2 = \mp 162s$, or, by transposition, $s^4 - 18s^2 = 81s^2 \mp 162s$. $\therefore\ s^4 - 18s^2 + 81 = 81s^2 \mp 162s + 81$, whence $s^2 - 9 = 9s \mp 9$ and $s = 9$ or $\frac{1}{2}(9 \pm \sqrt{153})$ ....(6). Substituting for s in (5) we get $m = 24$ or $\frac{1}{2}(51 \pm 9\sqrt{153})$.........(7).

Put $x + y = t$ and $xy = u$, and, neglecting the irrational values of s and m, we have $t + z = 9$, $u + tz = 24$ and $uz = 18$. Eliminating t and u from these three equations we get

$$z^3 - 9z^2 + 24z - 18 = 0 \ldots\ldots\ldots (8).$$

Put $z = v + 3$ and substitute in (8), we get $v^3 - 3v = 0$, $\therefore\ v^2 = 3$ and $v = \pm\sqrt{3}$. $\therefore\ z = 3 \pm \sqrt{3}$. But $x + y + z = 9$ (by (6)), $\therefore\ x + y = 9 - (3 \pm \sqrt{3}) = 6 \mp \sqrt{3}$. From (1) $xy = 18 \div z = 18 \div (3 \pm \sqrt{3},)$ $\therefore\ x^2 - 2xy + y^2 = (6 \mp \sqrt{3})^2 - 72 \div (3 \pm \sqrt{3}) = 3$. $\therefore\ x - y = \pm\sqrt{3}$. But $x + y = 6 \mp \sqrt{3}$ $\therefore\ 2x = 6$ and $2y = 6 \mp 2\sqrt{3}$, and consequently $x = 3$, $y = 3 \mp \sqrt{3}$ and $z = 3 \pm \sqrt{3}$.

35. "A man let a stone weighing 40 lbs. to a neighbor—the latter broke it accidentally into four parts—and upon returning the fragments consoled the owner by remarking that now he could weigh all numbers

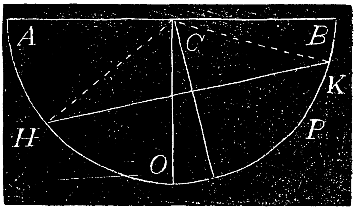
between one and forty. In other words, given $a + b + c + d = 40$, to determine such value for a, b, c and d as will, by *association*, produce all numbers from one to forty."

SOLUTION BY PROF. A. B. EVANS, LOCKPORT, N. Y.

It is evident that we must have $b - a = a + 1$, $c - b - a = a + b + 1$, $d - a - b - c = a + b + c + 1$. Therefore $b = 2a + 1$, $c = 2a + 2b + 1$, $d = 2a + 2b + 2c + 1$; and $a + b + c + d = 7a + 4b + 2c + 3 = 27a + 13 = 40$; $\therefore a = 1, b = 3, c = 9, d = 27$.

36. "A perfectly flexible cord of given length is suspended from two points whose coordinates are x', y' and x'', y''. How must the weight of the cord vary from point to point in order that it may hang in the arc of a circle."

SOLUTION BY WALTER SIVERLY, OIL CITY, PA.



Let H K be be the two given points of suspension, $\sqrt{(x' - x'')^2 + (y' - y'')^2} = 2b$ = the distance between them, $2l$ = the length of the cord, r the radius of the circle,
$$r\sin^{-1}\frac{b}{r} = l,$$
from which r may be determined and the circle drawn passing through H and K. Let AB be the horizontal diameter of the circle, O its lowest point to which transfer the origin making the axis of x horizontal and of y vertical, and let m be the unit of mass at any point P; and c the tension at O. It is shown in works on statics that

$$m = \frac{c}{g}\frac{d^2y}{dx^2} \div \frac{ds}{dx} \ldots\ldots\ldots\ldots\ldots\ldots\ldots(1).$$

The equation to the circle is,

$$x^2 = 2ry - y^2, \text{ or } y = r - \sqrt{(r^2 - x^2)},$$

$$\frac{dy}{dx} = \frac{x}{\sqrt{r^2 - x^2}}, \quad \frac{d^2y}{dx^2} = \frac{r^2}{\sqrt{(r^2 - x^2)^3}},$$

$$\frac{ds^2}{dx^2} = 1 + \frac{dy^2}{dx^2} = \frac{r^2}{r^2 - x^2}, \quad \frac{ds}{dx} = \frac{r}{\sqrt{r^2 - x^2}}.$$

Hence from (1), $m = \frac{c}{g}.\frac{r}{r^2 - x^2} = \frac{cr}{g(r - y)^2}$, or the unit of mass varies inversely as the square of its distance below the horizontal diameter of the circle.